DEVELOPMENT OF A SLIP LINE IN A BENDING BEAM

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In [1] the idea of an L-plastic material, namely, a material which satisfies the boundary conditions imposed on it not only by forming plastic regions, but also in slipping with respect to individual surfaces, is introduced. Variational formulations of the boundary value problem are considered in [2]. In this article the problem of the plane bending of a beam is solved within the framework of these formulations.

We will assume that in the deformed material there is a potential slip line which cuts the beam into two parts (Fig. 1). The position of the line and its shape is assumed to be known (the semiinverse formulation): The line L is a straight line passing through the center of the beam at an angle of $\pi/4$ to its longitudinal axis. We will assume that the material outside the slip line is deformed linearly elastically. Along the line we will assume the continuity of the normal component of the displacement vector

$$v_1 - u_1 = v_2 - u_2, \tag{1}$$

where v_1, v_2, u_1, u_2 are the components of the displacement vectors on the right and left regions of the line L (these regions will be denoted by D_2 and D_1). We will denote the discontinuity of the tangential component of the displacement vector (the slipping) by R:

$$R = (v_1 - u_1)\sqrt{2}/2 + (v_2 - u_2)\sqrt{2}/2.$$

We will assume that on these parts of the slip line where there is slipping, the tangential stress τ depends only on the value of the slipping:

$$\tau = f(R). \tag{2}$$

On parts where there is no slipping the line L does not function, and the elastic state of the material is preserved. Experimental data show that the curve f(R) can have a falling part [$f'(R) < 0$]. The development of the slip line for such materials can be both stable and unstable. By instability in the development we mean more increases in the slipping and extension of the slip parts for a small increase in the load parameter. Instability in the growth of the slip line appears as a certain increase in the slipping and the length of the slipping parts for a small increase in the load parameter. Dynamic effects which occur when there is a sudden development in the slip line from one stable state to another are not considered. We will assume that the surfaces of the beam $x_2 = \pm l/2$ are free from stresses; at the ends of the beam there are no tangential stresses, and the displacements normal to the surface are specified

$$\begin{aligned} u_1 &= -\Omega x_2 \text{ for } x_1 = -l, \ -l/2 \leqslant x_2 \leqslant l/2, \\ v_1 &= \Omega x_2 \text{ for } x_1 = l, \ -l/2 \leqslant x_2 \leqslant l/2, \end{aligned} \tag{3}$$

where x_1 and x_2 are Cartesian coordinates and Ω is the load parameter.

We will introduce the functional of the total "potential" energy [2]. For specified boundary conditions with respect to the stresses the functional takes the form

$$\Phi[u_1, u_2, v_1, v_2] = \frac{1}{2}E\int_{D_1} \psi_1 dx_1 dx_2 + \frac{1}{2}E\int_{D_2} \psi_2 dx_1 dx_2 - \int_L U ds, \tag{4}$$

where $\psi_1 = a_1(u_{1,1}^2 + u_{2,2}^2) + a_2 u_{1,1}u_{2,2} + a_3(u_{1,2} + u_{2,1})^2$; $\psi_2 = a_1(v_{1,1}^2 + v_{2,2}^2) + a_2 v_{1,1}v_{2,2} + a_3(v_{1,2} + v_{2,1})^2$; $a_1 = (1-\nu)/((1+\nu)(1-2\nu))$, $a_2 = 2\nu/((1+\nu)(1-2\nu))$, $a_3 = 1/(2+2\nu)$ in the case of plane deformation, $a_1 = 1/(1-\nu^2)$, $a_2 = 2\nu/(1-\nu^2)$, $a_3 = 1/(2+2\nu)$ in the case of the plane stressed state, $U = \int_0^R f(R)\,dR$ is the energy dissipation per unit length of the slip line, E is Young's modulus, and ν is Poisson's ratio. To solve this problem we will use the variational principle of the minimum of the total "potential" energy [2]. According to this principle the actual displacements yield a minimum of the functional (4) with respect to all the kinematically possible displacements (1) and (3).

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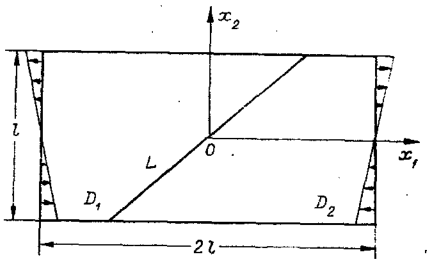

Fig. 1

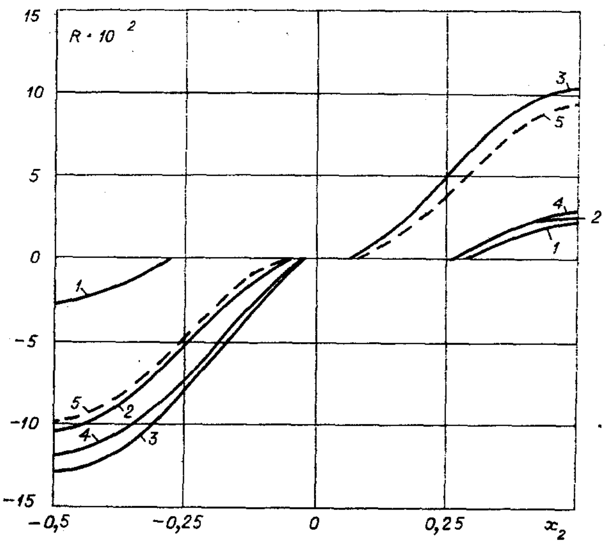

Fig. 2

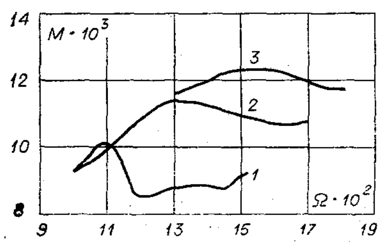

Fig. 3

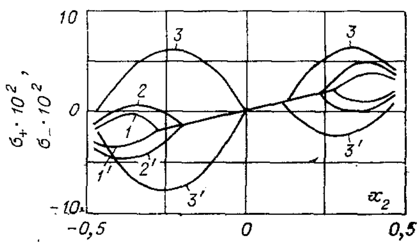

Fig. 4

TABLE 1

η	0,417	0,5	0,625	0,833
Ω_1	0,16	0,15	0,13	0,12
Ω_2	0,18	0,165	0,15	0,13

In this case the requirement that the functional Φ should be stationary for condition (1) ensures continuity on the line of possible slip of the normal and tangential component of the stress tensor. Hence, the problem reduces to minimizing the functional (4) with the continuity condition (1) and the boundary conditions (3). The problem can be solved numerically by direct minimization of the functional – by the method of local variations [3]. We will introduce the following dimensionless variables: $\sigma_{ij}l/(vE)$, R/v, x_1/l, x_2/l, u_i/v, v_i/v ($i, j = 1, 2$) (l is the width of the beam and v is the characteristic displacement), which we will denote in the same way as the dimensional variables. We will choose the quadratic difference grid in the region considered so that the line L intersects it only at junction points (the step of the grid Δx must fit a whole number of times into the section $l/2$). In this case, at each point of intersection of the grid with the slip line there are two grid junction points: One junction point belongs to the region D_1 and the other to the region D_2. As a result of this division we obtain triangular cells along the line L and square cells in the remaining region. We will denote the square cells by the symbol N_{ij} and the triangular cells of the region D_1 by the symbol P_{ij} and those of the region D_2 by the symbol Q_{ij} (i, j are the coordinates of the left lower vertex for this cell). The integrals (4) over the regions D_1 and D_2 are replaced by the sums

$$\int_{D_1} \psi_1 dx_1 dx_2 \approx \sum_{k,l} I_{kl}, \qquad \int_{D_2} \psi_2 dx_1 dx_2 \approx \sum_{m,n} J_{mn},$$

where $I_{kl} = H\psi_1((u_{p,q})_*)$ (p, q = 1, 2) is the value of the integral over the cell N_{kl} (P_{kl}) of the region D_1, $J_{mn} = H\psi_2((v_{p,q})_*$ (p, q = 1, 2) is the value of the integral over the cell $N_{mn}(Q_{mn})$ of the region D_2, $H = \begin{cases}(\Delta x)^2 \text{ for } N_{kl}(N_{mn}),\\ (\Delta x)^2/2 \text{ for } P_{kl}(Q_{mn})\end{cases}$ is the area of the cell, the asterisk denotes the taking of the average value of the corresponding quantity in a cell: $(u_{1,1})_* = (u_{1m+1n} - u_{1mn})/\Delta x$, $(u_{1,2})_* = (u_{1m+1n+1} - u_{1m+1n})/\Delta x$ for the cell Q_{mn}, $(u_{1,1})_* = (u_{1k+1l+1} - u_{1kl+1})/\Delta x$, $(u_{1,2})_* = (u_{1kl+1} - u_{1kl})/\Delta x$ for the cell P_{kl}, and $(u_{1.1})_* = (u_{1i+1j+1} - u_{1ij+1} + u_{1i+1ij})/(2\Delta x)$, $(u_{1.2})_* = (u_{1i+1j+1} - u_{1i+1j} + u_{1ij+1} - u_{1ij})/(2\Delta x)$ for the cell N_{ij}. The mean values of the derivatives of the components of the displacement vector, u_2, v_1, and v_2, are calculated in a similar way. The value of the energy dissipation along the slip line is approximated as follows:

$$\int_L U ds \approx \sum_{i,j} SU \ (R_{ij}),$$

where $S = \Delta x/\sqrt{2}$ at the extreme junction points of the line and $S = \Delta x\sqrt{2}$ at the remaining junction points; the quantity $U(R_{ij})$, where R_{ij} is the slip at the junction point with coordinates (i, j), is found from the diagram (2).

In order to reduce the value of the functional at each junction point the displacements u_{1ij}, u_{2ij}, v_{1mn}, v_{2mn} are varied with a fixed variation period. Along the slip line the variation in the components of the displacement vector obeys the continuity condition (1). For the initial value of the load parameter $\Omega^0 > 0$ before the beginning of the variation process the initial approximation (the zeroth) is specified. The iteration is assumed to be completed if at all junction points of the grating a further change in the components of the displacement vector with a specified variation period does not reduce the value of the functional. The variation along the specified grating is completed when the variation period is reduced to the specified value. Subdivision of the grating is used to accelerate the process of convergence. When changing from one grating to another the grating period Δx is reduced by a factor of 2. The initial approximation on the newly obtained finer grating is linearly interpolated for the new junction points of the solution from the previous grating. For the next value of the low parameter one specifies as the initial approximation the solution obtained on the previous step. Since the variation for a specified Ω begins on a rough grating, the solution from the previous step is first "matched" with the fine grating on the initial coarse grating. The number of load periods is determined by assigning a finite value of the load parameter Ω_k.

A program was compiled in FORTRAN using the above algorithm. The calculations were carried out on the BÉSM-6 computer. The dimensions of the initial grating were 5×11, junction points and the dimensions of the finer grating were 17×35 junction points.

The values of the parameters were chosen as follows: $\Omega^0 = 0 - 0.14$, $\Delta\Omega = 0.002 - 0.02$, $\Omega_k = 0.2$. The variation of the components of the displacement vector was discontinued as soon as the variation period became less than $2 \cdot 10^{-6}$. In this case the value of the functional was determined to the third significant figure. As a check version we solved the problem of the uniaxial extension of a beam with different f(R) diagrams, allowing strengthening and weakening sections. The solutions obtained differed from the accurate solution by not more than 0.8%. The solution of the problem of the bending of a beam for a strengthened material [$f'(R) > 0$] shows that slip regions occur at the ends of the line L and as Ω is increased they "penetrate" to the center of the beam. An increase in the length of the slip parts and of the slips R themselves with respect to the load parameter always occurs monotonically. We will consider in more detail the case when the material is weakened along the slip line; i.e., as the slip R increases the cohesive forces along the edges of the slip line decrease. We will confine ourselves to the linear approximation of the f(R) curve

$$f(R) = \begin{cases}\tau_s - \eta R & \text{for} \quad 0 < R \leqslant R_*,\\ 0 & \text{for} \quad R > R_*,\end{cases} \tag{5}$$

where $R_* = \tau_s/\eta$; $\eta > 0$ is the slope of the falling part; and τ_s is the dimensionless limit of the elasticity of the material for shear. Note that the solution algorithm considered can be applied for functions f of any form and for arbitrary values of the displacements. The calculations were made for the following values of the parameters: $\tau_s = 0.025$, $\nu = 0.3$, $v = 0.05$, $E = 1$, $l = 1$. The value of η was varied from 1.25 to 0.19. When changing to dimensional quantities this is equivalent to solving the problem with different slopes of the $\tau - R$ diagram, different elastic properties of the material, and different dimensions of the specimen. We will consider the deformation of a beam with an active load in successive steps $\Delta\Omega > 0$ with respect to the load parameter Ω from an initial value Ω^0 to a final value of Ω_k. The calculations show the following. When $0 < \Omega < \Omega_l$ the tangential stress on the line of possible slip does not exceed the limit τ_s, and the beam as a whole is deformed elastically ($\Omega_l = 0.1$ for the above-specified value of τ_s). So long as $\Omega > \Omega_l$, there will be slip regions at the ends of the slip line. When Ω is increased further the slip and the length of the slip regions increase stably and symmetrically with respect to the center of the beam. If the slope of the curve (5) is fairly small ($\eta < \eta_* = 0.375$) the slip line continues to develop stably and symmetrically as Ω increases up to Ω_k. If $\eta > \eta_*$, then for a certain value of $\Omega = \Omega_1$ the development of the slip line ceases to be stable: For a small increase in Ω from $\Omega = \Omega_1$ the increase in the length of one of the two slip parts occurs abruptly along the slip line. The length of the other part does not increase.

Hence, the symmetrical nature of the development of the slip line is disturbed; i.e., when an instability occurs in the development of the slip line the symmetry condition itself becomes unstable. A further increase in the load parameter $\Omega > \Omega_1$ causes an unstable increase in the slipping and in the length of the slip parts while preserving the nonsymmetry of the development of the line. However, for a certain value of $\Omega = \Omega_2$ the development of the slip line again acquires an unstable character: An abrupt increase in the length and in the slipping of the upper slip part of the line L occurs. As a result of this the jump in the length of the slip parts and of the slip which occurs in them are equalized. When Ω is increased from $\Omega = \Omega_2$ to $\Omega = \Omega_k$ the slip line keeps its stable and symmetrical form of development. Note that for values of η close to the critical value η_*, the values of the jumps in the slips and the jumps in the lengths of the slip parts are small, and the second jump in the development of the line may not be present. We noted above that when $\Omega > \Omega_1$ the symmetry in the increase in the slip parts is disturbed. We would expect however, that when $\Omega > \Omega_1$ it would be possible in principle for another form of development of the slip line to occur, when the jump occurs not on one but on both parts of the slip and the symmetry of the development of the line is not disturbed. Hence, the point $\Omega = \Omega_1$ is a bifurcation point of the solution; i.e., when $\Omega > \Omega_1$ the functional has two local minima separated, as calculation shows, by a fairly high "barrier." The solution for a certain load parameter $\Omega' > \Omega_1$ can be obtained by two methods. The first method consists in first solving the problem for a certain value of Ω^0 close to zero, then for $\Omega^0 + \Delta\Omega$ etc. As the initial approximation we always use the solution from the previous step. This method, in view of the calculation errors, which in this case play the role of small actual perturbations, always leads to a minimum corresponding to asymmetry of the development of the line. The second method consists in taking $\Omega' > \Omega_1$ immediately as the initial value of Ω^0. In this case the initial point (the zeroth approximation) lies "equally far" from both minima and we would therefore expect that the procedure which was followed in the iterations would also lead to a second minimum corresponding to the symmetrical solution. One of these solutions is shown in Fig. 2. It should be noted that in the actual loading process the value of the load parameter $\Omega = \Omega'$ is reached by a gradual increase in Ω from 0 to Ω' as $\Delta\Omega \to 0$. Hence, only the solution obtained by the first method – the asymmetrical solution – has actual meaning. The second (symmetrical) solution is only possible theoretically.

Figure 2 also shows curves of the distribution of the slips along the slip line for different Ω and $\eta = 0.625$. Curves 1-3 correspond to the case of asymmetrical development when $\Omega^0 = 0.1$ and $\Delta\Omega = 0.005$ and values of Ω = 0.125, 0.130, and 0.150, and curves 4 and 5 were obtained for the same value of $\Omega = 0.135$, but the first corresponds to asymmetrical development of the line ($\Omega^0 = 0.1, \Delta\Omega = 0.005$) and the second to symmetrical development ($\Omega^0 = 0.135$). The critical values of the load parameters Ω_1 and Ω_2 for different η are shown in Table 1. Figure 3 shows curves of the behavior of the supporting power (the value of the bending moment M) of the beam as a function of Ω. Curves 1-3 correspond to values of $\eta = 0.833$, 0.5, and 0.417. It is characteristic that as the load parameter is increased the supporting power first increases, despite the appearance of slip parts, and then begins to decrease. In this case the maximum value of the supporting power of the beam is reached when $\Omega_l < \Omega < \Omega_1$, i.e., in the state of incipient slip along the line and until the first slip jump. A somewhat unexpected result is observed in the behavior of the tangential stress along the edges of the slip parts (the components of the stress tensor σ_{mm}, where the vector **m** is directed along L). The tangential stress on the left edge of the slip line is denoted by σ_- and on the right by σ_+. Curves 1'-3' of Fig. 4 correspond to the value σ_- and curves 1-3 to the value σ_+. Consider the lower half of the beam. So long as the beam is in the elastic state all the components of the stress tensor on the slip line are continuous. When the line becomes activized (when slip occurs) discontinuities in the tangential stresses occur at the same time (curves 1 and 1' of Fig. 4 for Ω = 0.12 and $\eta = 0.5$). For small Ω the stresses σ_- and σ_+ are contracting.

This is natural since contracting forces act on the lower half of the ends of the beam. However, when the load parameter is increased further the value of σ_+ becomes positive – a stretching region occurs due to slip along the line L (curve 2, Fig. 4, $\Omega = 0.13$ and $\eta = 0.5$). Similar behavior of the stresses is observed in the upper half of the beam (see Fig. 4). Note that the variational formulations of the boundary-value problems [2] and the algorithm considered can also be used to solve problems on the development of cracks in normal fracture.

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LITERATURE CITED

1. A. F. Reveuzhenko, S. B. Stazhevskii, and E. I. Shemyakin, "Asymmetry of plastic flow in a converging symmetrical channel," FTPRPI, No. 3 (1977).
2. A. F. Revuzhenko and E. I. Shemyakin, "Some formulations of boundary-value problems of L-plasticity," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1979).
3. F. L. Chernous'ko and N. V. Banichuk, Variational Problems in Mechanics and Control [in Russian], Nauka, Moscow (1973).

NUMERICAL SIMULATION ON A COMPUTER OF THE PROCESS OF EXPLOSIVE FORMING

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The problem of simulating the dynamic behavior of an axisymmetrical blank for explosive forming under plane stressed state conditions with rigid fastening or hinged rest on a contour is considered in a number of papers [1-4] (a detailed bibliography is given in [5]), and in [6] a method is described for determining the dynamic behavior of thin nonaxisymmetrical shells of ideally plastic material deformationally hardened and sensitive to the rate of deformation for the boundary conditions described above. In this paper we describe a method for the numerical calculation of the dynamic behavior of nonaxisymmetrical blanks of complex configuration. Unlike publications where problems unrelated to practice are solved, here we simulate the process of forming–drawing, taking into account both the displacement of the flange part of the blank and the forces of friction which occur on the flange part of the blank during high-speed deformation. In addition, as a result of an optimizational search the optimum external load applied to the blank is determined, which enables the values and positions of the charges required to deform it to be found.

1. The system of differential equations describing the motion of a blank (more accurately a Lagrange network, connected with its middle surface) can be written exactly as in [5] and can be solved in explicit form using the method of finite differences [7]. It turns out that the finite-difference model is sensitive to the integration step in time. In addition, the stability of the difference scheme depends on the initial value of the cell of the integration network. According to [8], the upper boundary of the integration step in time is expressed in the form

$$\Delta t = 2/\omega_{\max},$$

where $\omega_{\max}$ is the highest eigenfrequency of the corresponding finite-difference model. However, solving problems it is extremely inconvenient to determine in advance the frequency $\omega_{\max}$ corresponding to each specific finite-difference model. Hence, to determine Δt one can use the condition [5]

$$\Delta t \leqslant \Delta X_{\min}(\rho(1-\nu^2)/E)^{1/2},$$

where $\Delta X_{\min}$ is the value of the cell of the network, ρ is the density of the material of the blank, ν is Poisson's ratio, and E is Young's modulus. When integrating the equations at each subsequent instant of time one determines the displacement of the junction points of the Lagrange network. If at the initial instant of time t = 0 we write the equation of motion

$$F^j_{mn} = \bar{\rho}\ddot{Y}^j_{mn},$$

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